1 SEM TDC MTMH (CBCS) C1

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(March)

MATHEMATICS

(Core)

Paper : C–1

(Calculus)

Full Marks : 60

Pass Marks : 24

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. (a) Write the value of $\frac{d}{dt}(\tanh t)$. 1

(b) Write the value of $\frac{d^n}{dx^n}(\sin ax)$. 1

(c) Write intervals in which $y \quad x^3$ is concave up and concave down. 2

(d) Determine the concavity of $y \quad 2 \quad \sin x$ on $[0, 2 \quad]$. 2

(e) Show that $\cosh 2x \quad \cosh^2 x \quad \sinh^2 x$ 3

Or

Show that $\cosh^{\ 1} \frac{1}{x} \quad \mathrm{sech}^{\ 1} x$

(f) Find y_n (any one) if—

(i) $y \quad \cos^3 x$;

(ii) $y \quad \frac{a \quad x}{a \quad x}$. 3

(g) If $y \quad \tan^{\ 1} \frac{x}{a}$, then find y_n. 4

Or

If $\log y \quad \tan^{\ 1} x$, show that

$$(1 \quad x^2) y_2 \quad (2x \quad 1) y_1 \quad 0$$

(h) Evaluate : 4

$$\lim_{x \quad 0} \frac{x \quad \sin x \cos x}{x^3}$$

Or

Find the asymptote of the curve

$$y^3 \quad x^2 y \quad 2y^2 \quad 4y \quad 1 \quad 0$$

2. *(a)* Evaluate : 2

$$ _0^{\overline{2}}\cos^3 x\,dx $$

(b) Evaluate (any *one*) : 4

(i) $_0^{\overline{2}}\sin^4 x\cos^5 x\,dx$

(ii) $\sec^6 x\,dx$

(c) Obtain the reduction formula for

$$\tan^n x\,dx$$ 4

Or

A region is enclosed by the triangles with vertices (0, 1), (1, 0), (1, 1). Find the volume of the solid generated by revolving the region about the *y*-axis.

(d) The circle $x^2 \quad y^2 \quad a^2$ revolves round the *x*-axis. Find the volume so generated. 5

Or

A region bounded by the curve $y \quad \sqrt{x}$, the *x*-axis, and the line $x \quad 4$ is revolved about the *x*-axis to generate a solid. Find the volume of the solid.

3. *(a)* Write the parametric formula for $\frac{dy}{dx}$. 1

(b) Write the equation of the circle in polar form. 1

(c) Write the equivalent Cartesian equation of $r^2 \sin 2 \quad 2$. 2

(d) The position of a particle moving in the *xy*-plane is given by $x \quad \sqrt{t}$, $y \quad t$. Find the path traced out by the particle. 2

(e) Find a parametrization for the curve having the line segment with end points (–1, –3) and (4, 1). 4

Or

Parametric equations and parameter interval for the motion of a particle in *xy*-plane is given $x \quad \cos 2t$, $y \quad \sin 2t$, $0 \quad t \quad$. Identify the particle's path by finding a Cartesian equation.

(f) Find the length of the astroid

$$x \quad \cos^3 t,\ y \quad \sin^3 t,\ 0 \quad t \quad 2$$ 5

Or

Find the area of the surfaces generated by revolving the curve $x \quad \cos t$, $y \quad 2 \quad \sin t$, $0 \quad t \quad 2$ about *x*-axis.

4. *(a)* Write the necessary condition for the vectors $\vec{a}, \vec{b}, \vec{c}$ to be co-planar. 1

(b) The position vector of a moving particle is given by

$$\vec{r} \quad \cos 2t\hat{i} \quad 2\sin 4t\hat{j} \quad t^2\hat{k}$$

Find the acceleration at any time *t*. 2

(c) Find the volume of the parallelopiped whose edges are represented by

$\vec{a} \quad 2\hat{i} \quad 3\hat{j} \quad 4\hat{k},\ \vec{b} \quad \hat{i} \quad 2\hat{j} \quad \hat{k},\ \vec{c} \quad 3\hat{i} \quad \hat{j} \quad 2\hat{k}$ 3

Or

Let

$$\vec{R}(u) \quad 4\hat{i} \quad (u^2 \quad 6u^3)\hat{j} \quad u^2\hat{k}$$

Find $_{1}^{3}\vec{R}(u)\,du$.

(d) Evaluate $\vec{a} \quad (\vec{b} \quad \vec{c})$, where

$\vec{a} \quad \hat{i} \quad 2\hat{j} \quad 3\hat{k},\ \vec{b} \quad 2\hat{i} \quad \hat{j} \quad \hat{k},\ \vec{c} \quad \hat{i} \quad 3\hat{j} \quad 2\hat{k}$ 4

Or

Find the tangential component of acceleration of a moving particle.

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