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1 SEM TDC MTMH (CBCS) C1

2019

(December)

MATHEMATICS

(Core)

Paper : C–1

(Calculus)

Full Marks : 60

Pass Marks : 24

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. (a) Write the value of $\frac{d}{dx}(\cosh x)$. 1

(b) Let $f(x)$ is a differentiable function on an open interval I. Write when it is concave up. 1

(c) Find $\frac{d}{dx}(\tanh\sqrt{1+x^2})$. 2

(d) Find y_n (any *one*) : 3

(i) $y = \sin^2 x \cos^2 x$

(ii) $y = \sin x \sin 2x \sin 3x$

(e) If $y = x^2 \tan^{-1} x$, then find y_n. 4

Or

If $y = (\sin^{-1} x)^2$, then show that

$$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2 y_n = 0$$

(f) If $y = e^{ax+b} \sin x$, then find y_n. 3

(g) Evaluate (any *one*) : 3

(i) $\lim_{x \to 0} \frac{\tan x - x}{x - \sin x}$

(ii) $\lim_{x \to \infty} \frac{x^2 + 3x}{1 - 5x^2}$

(h) Find the minimum value of the function

$$f(x) = 1 + 2\sin x + 3\cos^2 x, \ x \in \left[0, \frac{\pi}{2}\right]$$ 3

2. (a) Show that

$$\int_0^{\frac{\pi}{2}} \sin^n x \, dx = \int_0^{\frac{\pi}{2}} \cos^n x \, dx$$ 1

(b) Obtain the reduction formula for

$$\int \sin^n x \, dx, \ n > 1$$

and hence write the value of $\int \sin^3 x \, dx$.

4+1=5

(c) Evaluate (any *one*) : 4

(i) $\int \tan^5 x \, dx$

(ii) $\int \sec^5 x \, dx$

(d) A region bounded by the curve $y = x^2 + 1$ and the line $x + y = 3$ is revolved about the x-axis to generate a solid. Find the volume of the solid. 5

Or

A region is enclosed by the triangle with vertices (1, 0), (2, 1), (1, 1). Find the volume of the solid generated by revolving the region about the y-axis.

3. (a) Write the equation of parabola in polar form. 1

(b) Find an equation for the hyperbola with eccentricity $\frac{5}{3}$ and directrix $x = 3$ in polar form. 2

(c) Find the parametric equations and a parameter interval for the motion of a particle that starts at $(a, 0)$ and traces the circle $x^2 + y^2 = a^2$, once clockwise. 2

(d) Determine the nature of the conic represented by

$$x^2 + 2xy + y^2 + 2x - y + 2 = 0$$ 2

(e) Determine the angle of rotation of axes in order to remove the xy term from

$$3x^2 - 2\sqrt{3}xy + y^2 = 1$$ 2

(f) Find a parameterization for the line segment with end points $(-1, -3)$ and $(4, 1)$. 3

Or

Find perimeter of a circle of radius a defined parametrically by $x = a\cos t$, $y = a\sin t$, $0 \le t \le 2\pi$.

(g) Find the area enclosed by the ellipse $x = a\cos t$, $y = b\sin t$, $0 \le t \le 2\pi$. 3

Or

Find the area of the surface generated by revolving the curves $x = \cos t$, $y = 2 + \sin t$, $0 \le t \le 2\pi$, about x-axis.

4. *(a)* Let $\vec{r}(t) = (\cos t)\hat{i} + (\tan t)\hat{j} + t\hat{k}$. Find

$$\lim_{t \to \frac{\pi}{4}} \vec{r}(t)$$ 2

(b) If $\vec{r}(t) = (4\cos t)\hat{i} + (4\sin t)\hat{j} + (\cos^2 t)\hat{k}$ be the position vector of a particle at any time t, then find velocity at any time t. 2

(c) Let $\vec{r}(t)$ is a differentiable vector function of t of constant length. Show that $\vec{r}(t)$ and $\vec{r}\,'(t)$ are orthogonal. 2

(d) Evaluate

$$\int_0^{\pi}[(\cos t)\hat{i} + 2t\hat{j} + 3t^2\hat{k}]dt$$ 3

Or

Find the scalar triple product of the vectors $\hat{i}+2\hat{j}+3\hat{k}$, $\hat{i}+3\hat{j}+5\hat{k}$ and $\hat{i}+\hat{j}+6\hat{k}$.

(e) Write the value of $\vec{a}\times(\vec{b}\times\vec{c})$. 1

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