General Properties of Nuclei:-

Chapt. - 1

(A) Constituents of Nucleus:-

In 1932, nutron was discovered by Sadwick.

→ Proton - Electron hypothesis:-

Befor the discovery of nutron it was assum that nucleus consist of proton & electrons

According to this model nucleus consists of Z no of protons and (A-Z) no. of electrons. Such that total charge of the nucleus is +ze and Z no of e^- will revolve around the nucleus so that atom is electrically neutral.

Total charge of the atom is zero. So that atom is electrically nutral.

(2) During β-decay electrons are emmited from nucleus and this supports the proton-electron hypothesis.

Failure of P-e hypothesis :-

① Non - existance of e inside the nucleus :-

According to heisenbarg uncertainity principle

$\Delta P \Delta x = \hbar$

$\Rightarrow \Delta P = \frac{\hbar}{\Delta x}$

If electron exists inside nucleus then uncertainty in the position $\Delta x = 2 \times 10^{-14}$ m (radius)

$\therefore \Delta P = \frac{\hbar}{2\times 10^{-14}}$ $\left(\hbar = \frac{h}{2\pi} \right)$

$\Rightarrow \Delta P = \frac{h}{2\pi \times 2 \times 10^{-14}}$

$= \frac{6.67 \times 10^{-34}}{2 \times 3.14 \times 2 \times 10^{-14}}$

$= \frac{1.667 \times 10^{-20}}{3.14}$

$\Rightarrow \Delta P = 5.278 \times 10^{-21}$ Kg m/s.

$\therefore$ minimum uncertainity is 5.278×10^{-21} kg.

The minimum momentum of the electron can be taken as $P_{min} = 5.278 \times 10^{-21}$ Kg m/s

The minimum energy of the electron can be calculated using the relativistic energy eqn

$E_{min}^2 = P^2C^2 + m_0^2 C^4$

$$\Rightarrow E_{min} = \sqrt{p^2c^2 + m_0^2c^4}$$

$$= 9.87$$

For electron to be preresent in the nucleus. It should have energy at least around 10MeV. Experimentally it's founded the maximum energy of the electrons/positrons during $\beta-$ decay carry energy '2-3' MeV. This concluds that electrons can't reside inside the nucleus.

② Proton can exist inside nucleus :-

According to heisenburg uncertainity principle

$$\Rightarrow \Delta P \Delta x = \hbar$$

$$\Rightarrow \Delta P = \frac{h}{2\pi \times \Delta x}$$

Since the position Δx (radius) $= 2 \times 10^{-14}$

$$\Rightarrow \Delta P = 5.278 \times 10^{-21} \text{ Kg m/s}$$

The minimum energy of proton can be taken as

$$P_{min} = 5.278 \times 10^{-14} \text{ kg. m/s}$$

- The minimum energy of the proton can be cal- -culated using the relativistic energy eqn

$$E^2 = \frac{p^2}{2m}$$

② The experimental result shows that the magnetic moment associated with the nucleus is of the order of nuclear magneton $\left(\mu_N = \frac{e\hbar}{2m_p}\right)$ not in the order of Bohr magneton. $\left(\mu_B = \frac{e\hbar}{2m_e}\right)$. This rules out the presence of electron inside the nucleus.

③ Some of the electrons of the atom reside inside the nucleus and some of the e^- are revolving around the nucleus is quite contradictory.

The De Broglie wavelength of e^- is found to be greater than the dimension of the nucleus.

④ Proton Neutron Model of Nucleus:-

After discovery of Neutron in 1932 it was establish that nucleus consist of protons and neutrons.

Nucleus is represented by $^{A}_{Z}X$, where A is mass no. and Z is no. of proton.

Atomic mass unit defined as $\frac{1}{12}$ of one $^{12}_{6}C$

$$1u = \frac{1}{12} \times \text{mass of one carbon } ^{12}_{6}C$$

$$= \frac{1}{12} \times \frac{12}{6.022 \times 10^{23}} \times 10^{-3} \text{ gm}$$

$$\Rightarrow 1u = 1.66 \times 10^{-27} \text{ kg}.$$

Equivalent mass of 1U :-

$E = mc^2$

$= 1U \times C^2$

$\Rightarrow E = 1.66 \times 10^{-27}\ kg \times (3 \times 10^8\ m/s^1)^2$

$= 4.98 \times 10^{-19}\ J$

Converting in to ev

$\Rightarrow E = 931.5\ MeV.$

i.e. 931.5 MeV is realesed when 1U mass disappear.

Binding Energy :-

It is the energy required to separted the nucleon from the nucleus so that there will be no interaction between them.

Or

The sum of mass of proton & neutrons of the nucleus is always greater then the actual mass of the nucleus. These disappeared mass is converted into energy which will bound the nucleons inside the nucleus. This is called binding energy of the nucleus.

Binding Energy per nucleon :-

This is the energy required to seperate a nucleon from the nucleus. $E_{bn} = \dfrac{\text{Binding Energy}}{A}$

(A) Mass defect & Binding Energy:-

Mass deffect = The sum of mass of proton & Netron − Actual mass of the nucleus.

∴ Binding Energy (B.E) = $\Delta m \times c^2$

$$\Rightarrow B.E = \left[Z m_p + (A-Z) m_n - M_{nuc}\left({}^{A}_{Z}X\right)\right] c^2$$

$$\Rightarrow B.E \text{ for atomic mass} = \left[Z m_p + (A-Z) m_n - {}^{A}_{Z}X\right] c^2$$

$$\Rightarrow B.E = \left[Z m_p + (A-Z) m_n - (M_{atomic} - Z m_e)\right] c^2$$

Q.1:- Calculate the binding energy of alpha particle given the atomic mass of ${}^{4}_{2}He$ is 4.0026033 u

Solⁿ:- given ${}^{4}_{2}He$ = 4.002603 u

Therefore the binding energy will be

$$\Rightarrow B.E. = \left[Z m_p + (A-2) m_n - (M_{atomic} - Z m_e)\right] c^2$$

$$\Rightarrow B.E = \left[2 \times 1.007276 + 2 \times 1.00866 - (4.002603 - 2 \times 9.1 \times 10^{-31})\right] \times 3 \times 10^{8} \times 931.5 \text{ MeV}$$

$$\Rightarrow B.E = \left[2.014552 + 2.01733 - 2.182603 \times 10^{-30}\right] \times 2.79 \times 5 \times 10^{8}$$

B.E = 5. . . MeV.

⊛ B.E & stability relation:-

More is the binding energy greater is the stability.

⊛ Binding energy per nucleon v/s mass no. graph:-

The nucleus that Even no. of N and even no. of Z are more stable.

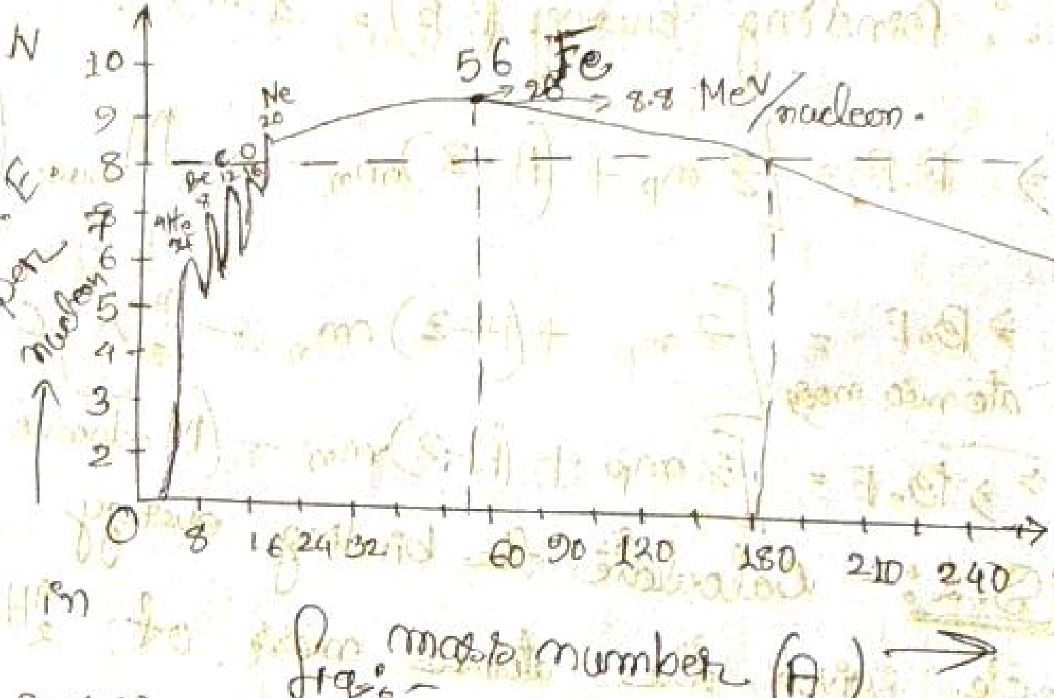


fig:-

① The sudden jump in the binding energy per nucleon establish that those nuclei (^4_2He, ^8_4Be, $^{12}_6C$, $^{16}_8O$, $^{20}_{10}Ne$) have more binding energy. ie More stability than the surrounding nuclei this is because all those nuclei have even no of protons and even no. of nuetrons.

② $^{56}_{26}Fe$ has the maximum binding energy per nucleon. (8.8 MeV Per nucleon).

(3) Two remarkable conclusion can be drawn ~~by~~ from the graph.

(i) If we can split a heavy nucleus into two medium sized nuclei, each of the nuclei will have more binding energy per nucleon than the original nucleus. The extra energy will be realesed & it's a huge amount of ~~of~~ energy. This process is called nuclear "fission".

(ii) By joining two light nuclei together and forming a single nucleus also realesed energy & that process is called nuclear "fussion".

(4) N - A plot [N - Z plot] :-

From N-Z graph it's clear that for A<20 no of protons are approximatly equal to no. of nutrons, while in heavier nuclei the no. of nutrons becomes greater than the no of protons.

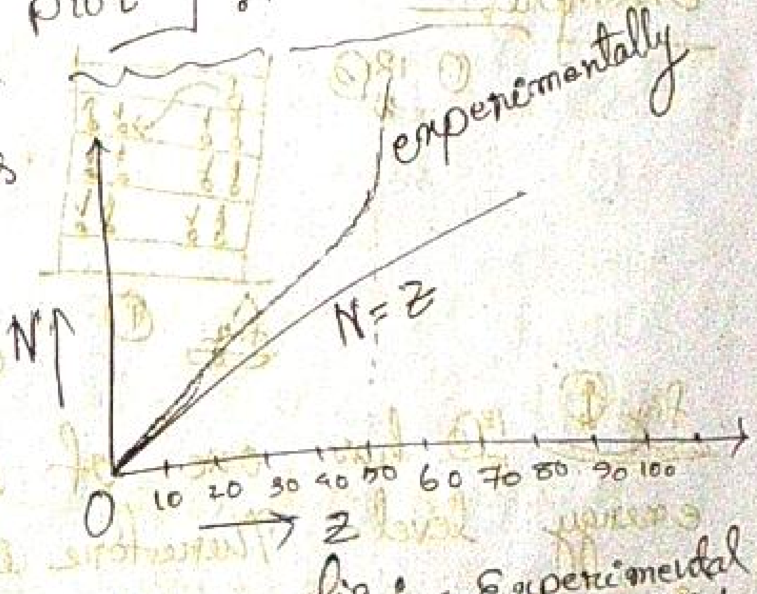


fig :- Experimental graph expected.

Reason :-

Protons are +ve charged and repal one another electrically. The repalsion becomes so great in nuclei with more than 10 protons, such that access nutrons are accomulated in order to overcome the repalsion force to obtain stability.

Nucleons are fermions (spin $\frac{1}{2}$) & they obey pauli-exclusion prenciple. So that each nuclear energy level can contain two nutrons of opposite spins. Energy levels are filled in sequence to achive the configuration of minimum energy & more stability.

Example:

(i) $^{12}_{5}B$

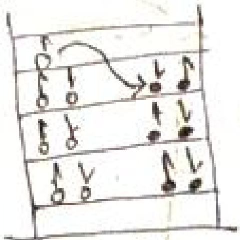
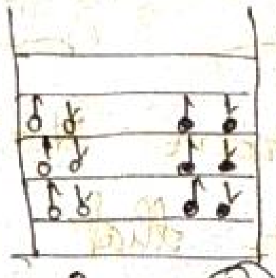


fig (I)

(ii) $^{12}_{6}C$



configuration fig (II)

fig (I) $^{12}_{5}B$ has one of its nutron in higher energy level. Therefore it's unstable compared to $^{12}_{6}C$. If somehow $^{12}_{5}B$ is created in nuclear reaction, it will imediatly changed to $^{12}_{6}C$ by under going β-decay. $(n \longrightarrow p)$

Fig ② - $^{12}_{6}C$ has the configuration of minimum energy and maximum stability with each completely filled energy levels compared to $^{12}_{5}B$.

④ Nuclear size Determination:-

① The high energy e- beam are used as a probe to determine the size of the nucleus.

② Relativistic total energy of electron

$$E = \sqrt{p^2c^2 + m_0^2c^4}$$

& Relativistic K.E $= \sqrt{p^2c^2 + m_0^2c^4} - m_0c^2$

where m_0c^2 is rest mass energy. m_0c^2 (of e^-) = 511 KeV

If K.E of the electron K.E $\gg m_0c^2$,

in that case for high momentum →

The total energy $E = K.E = PC$.

Using De Broglie relation the wavelength of the electron beam $\lambda = \frac{h}{p} = \frac{hc}{pc} = \frac{hc}{K.E} = \frac{hc}{E}$

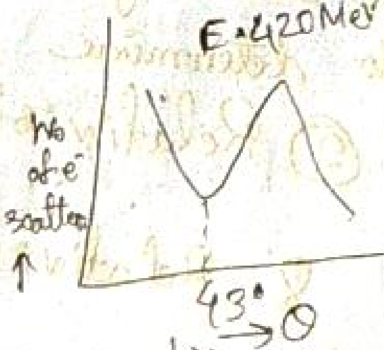
$$\Rightarrow \lambda = \frac{1240\ MeV\ Fm}{E}$$

where E is K.E.

In order to get information about the nucleus the energy of the e^- must be equal to the order 10^2 MeV. This is the scattering graph of e^- from target nucleus O_2. Considering the diffrac-tion of e^- beam from the circular apparature nucleus, the first order minimum occurs at $\sin\theta = \frac{1.22\lambda}{D}$. D is diameter of apparature.

So given energy $E = 420$ MeV

$$\therefore \lambda = \frac{1240 \text{ MeV fm}}{420 \text{ MeV}}$$



$\theta = 43°$, $\lambda \cong 3$ fm.

$$\sin 43° = \frac{1.22 \times 3}{D}$$

$$\Rightarrow D = 5.2 \text{ fm}.$$

from here rough estimation of diameter of big O_2 nucleus comes out to be 5.2 fm and radius will be 2.6 fm.

Nuclear size determination:-

(A) Nuclear charge density and Mass density:-

(1) Charge density Or proton density :-

from experimental graph (1) and (2) it is clear that density is almost constant from the centre of the nucleus to some distance and gradually it decreases.

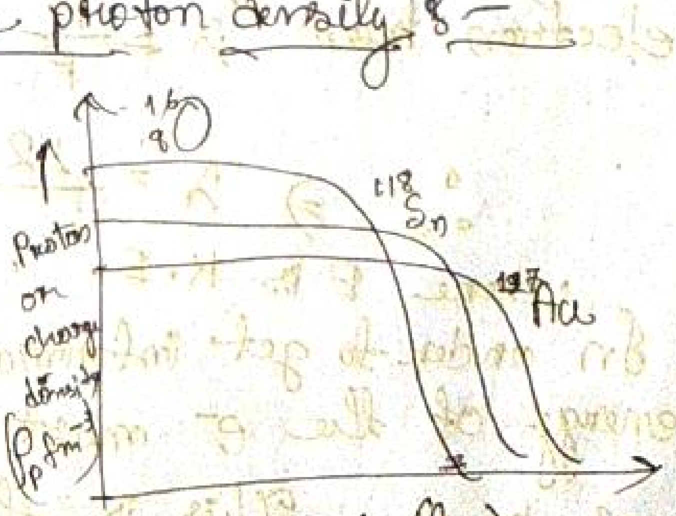


fig:- (1)

This type of graph can be describe by wood - saxon function i.e. $\rho = \frac{\rho_0}{1+e^{\left(\frac{r-R}{a}\right)}}$

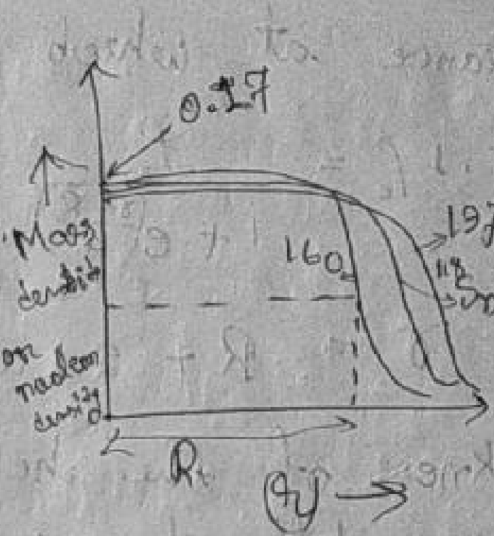


fig ② experimental graph.

here ρ_0 is charge const.

r = distance

R = distance at which density becomes $\rho_0/2$.

a = is related to how sharply the graph fall. it's the thickness of all shell where density ρ falls from $0.9\rho_0$ to $0.1\rho_0$.

For almost all nuclei ρ_0 is found to be $0.17\ fm^{-3}$.

Density at which density become $0.9\rho_0$.

$$\Rightarrow 0.9\rho_0 = \frac{\rho_0}{1+e^{\left(\frac{r-R}{a}\right)}}$$

$$\Rightarrow \frac{1}{0.9} = 1 + e^{\left(\frac{r-R}{a}\right)}$$

$$\Rightarrow 1.1 = 1 + e^{\frac{r-R}{a}}$$

$$\Rightarrow r = R - 2.3a.$$

distance at which density become $0.1\rho_0$

$$\Rightarrow 0.1\rho_0 = \frac{\rho_0}{1+e^{\left(\frac{r-R}{a}\right)}}$$

$$\Rightarrow r = R + 2.1a.$$

Thickness of the shell is $= 4.4a$

Experimentally it was found that $a \simeq 0.5$ fm for all nuclei. The value of R can be written as $R = R_0 A^{1/3}$

Here $R_0 = 1.2$ fm and A is the mass no.

Nucleus doesnot have any sharp boundary.

2/ (A) Muonic X-Ray Method :-

Muons are the particles of e^- family & these are observed in cosmic rays and can be produced in laboratory in high energic reactions. μ^+ & μ^- have the same charge as of e^+ & e^- and mass of muon's is about $207\ M_e$.

When a beam of μ^- is passed through matter (atom) muon's are captured in e^- orbits around the nuclei and those atomes are called muonic atoms.

The radaie of the muonic orbit are much

smaller than the e^- orbit by a factor.

$$\frac{m_e}{m_\mu} = \frac{1}{207}$$

For eg :- K orbit of muonic gold (Au) atom has radius of $a_0 = \frac{m_e}{m_\mu} \times 0.529 \text{Å}$

$$\Rightarrow a_0 = 3.23 \times 10^{-15} \text{ m}$$

$$\Rightarrow a_0 = 3.23 \text{ fm}.$$

The radius of Au nuclei is $R = R_0 A^{1/3}$

$$= 1.2 \times (197)^{1/3}$$

$$= 7 \text{ fm}.$$

Thus the muonic K orbit of Au atom is expected to lie wholly inside the nucleus for heavy atom.

From muonic X-Ray the radius of the nucleus can be approximated and it is found that the value is almost. $r_0 = (1.15 \pm 0.03)$ fm.

3/ Mirror Nuclei Methode :-

Using mirror nuclei methode radius of nuclei can be approximated on the basis of separation in culomb energy.

Considering the spherical shape of the nucleus charge density $\rho = \frac{Ze}{\frac{4}{3}\pi R^3}$

Considering a spherical core of thickness dx from a distance x from the centre of the nucleas. The charge on the spherical core,

$$q_1 = 4/3 \pi x^3 \rho$$

The charge on the shell q_2 is

$$q_2 = 4\pi x^2 \times dx \times \rho$$

The electrostatic potential bet^n the core & the cell is

$$= \frac{q_1 q_2}{4\pi\varepsilon_0 x} = \frac{4/3 \pi x^3 \rho \times 4\pi x^2 \rho\, dx}{4\pi\varepsilon_0 x}$$

$$= \frac{4/3 \pi x^4 \rho^2 dx}{4\varepsilon_0 \pi}$$

$$= \frac{1}{3\varepsilon_0} 4 \times \pi^2 \times x^4 \times \rho^2 \times dx$$

$$= \frac{16\pi^2 x^4 \rho^2 dx}{4.3\varepsilon_0 \pi}$$

Total electrostatic potential energy of the nucleas

$$\Rightarrow E_c = \int_0^R \frac{1}{4\pi\varepsilon_0} \frac{16}{3} \pi^2 x^4 \rho^2 \, dx$$

$$\Rightarrow E_c = \frac{1}{4\pi\varepsilon_0} \frac{16}{3} \rho^2 \pi^2 \frac{R^5}{5}$$

$$\Rightarrow E_c = \frac{1}{4\pi\varepsilon_0} \frac{16}{3} \pi^2 \frac{Z^2 e^2}{\frac{16}{9} \pi^2 R^6} \times \frac{R^5}{5}$$

$$\boxed{\Rightarrow E_C = \frac{1}{4\pi\epsilon_0}\frac{3}{5}\frac{Z^2e^2}{R}}$$

The separation bet^n two mirror nuclei (culomb energy) is

$$\Rightarrow \Delta E_c = E_{C_2} - E_{C_1}$$

$$= \frac{1}{4\pi\epsilon_0}\frac{3}{5}\frac{(Z+1)^2 e^2}{R} - \frac{1}{4\pi\epsilon_0}\frac{3}{5}\frac{Z^2e^2}{R}$$

$$\Rightarrow \Delta E_c = \frac{1}{4\pi\epsilon_0}\frac{3e^2}{5R}\left(\cancel{Z^2} + 2Z + 1 - \cancel{Z^2}\right)$$

$$\Rightarrow \Delta E_c = \frac{1}{4\pi\epsilon_0}\frac{3e^2}{5R}(2Z+1)$$

$$\boxed{\Rightarrow \Delta E_c = \frac{1}{4\pi\epsilon_0}\frac{3e^2}{5R} A}$$

This is the equation separation in culomb energy. From this separation energy, the radius of the mirror nuclei can be estimated.

These are three method to determine nucl-ear size.

(4) Nuclear Spin :-

The spin of a nucleus is the resultant of it's constituent nucleon's proton and nutrons.

The spin of protons & nutrons can be repres-ented by like that of e^-, ie $\frac{1}{2}\hbar$.

The orientation of the spin vector is such that the component of spin can be either parallel or antiparallel to a specific direction.

In addition to the spin angular momentum the protons & nutrons in a nucleas have orbital angular momentum. Thus the total intrinsic angular momentum of a nucleas is $I = \Sigma l_n + \Sigma l_s$.

$$\Rightarrow \vec{I} = \vec{L} + \vec{S}$$

here Σl_s is contribution of spin and Σl_n is the contribution of orbital angular momentum of all the nucleons.

The total angular momentum of the nucleas is given by $J_N = \sqrt{I(I+1)}\hbar$

The spin of nucleas can be experimentaly determine by serveral methods. It is experimantely found that.

① A nucleas with odd mass no has half integral spin. [ie. $\frac{1}{2}\hbar$, $\frac{3}{2}\hbar$, $\frac{5}{2}\hbar$, ...]

And a nucleas with even mass no has integral spin of [$0\hbar$, $1\hbar$, $2\hbar$, ...].

Intrinsic angular momentum

(3) For ground state of even Z(proton) & N(netron) . I = 0.

The total angular momentum I of a nucleus is usually called the spin of the nucleus or nuclear spin.

(A) Parity :-

In Quantum mechanics nuclear particle is described by a wave function $\psi(x,y,z)$ which depends on space coordinates x, y, z and if ψ^* is the complex conjugate of ψ then $\psi^*\psi = |\psi|^2$ gives the probability of finding the particle at any given point.

Parity is the property of such a wave f^n representing a quantum mechanical nuclear state, which may or may not change its sign on inversion of the space coordinate from (x, y, z) to (-x, -y, -z).

i.e. On reflection of the coordinate system at the origin. The parity of nucleus is thus related the behavior of nuclear wave f^n as a resultant of reflection. If $\psi_{(-x,-y,-z)} = \psi_{(x,y,z)}$ - It is known as even or +ve parity.

And if $\psi_{(-x,-y,-z)} = -\psi_{(x,y,z)}$ then odd or -ve parity.

The parity of a nucleus is determined by the $(-1)^l$, 'l' is the orbital angular momentum.

(A) Nuclear Magnetic Moment :-

From electromagnetism, a current carrying loop behaves like a magnetic dipole, with dipole moment $\mu = ia$.

If the current is caused by charge 'e' moving with velocity v in a circle of radius r then

$$|\mu| = iA$$

$$= \frac{e}{T} \times \pi r^2$$

$$= \frac{e}{\frac{2\pi r}{v}} \times \pi r^2$$

$$= \frac{evr}{2}$$

$$= \frac{emvr}{2m}$$

$$\Rightarrow |\mu| = \frac{e}{2m} |\vec{l}|, \quad \vec{l} \text{ is angular momentum.}$$

$$\Rightarrow |\mu| = \frac{e}{2m} l\hbar \quad \text{--- (1)}$$

$\frac{e\hbar}{2m}$ is called magneton.

$\Rightarrow$ for electron

$$\mu_B = \frac{e\hbar}{2m_e} = 5.7884 \times 10^{-5}\ eV/T$$

⇒ for nuclear magneton

$$\mu_N = \frac{e\hbar}{2m_p} = 3.1525 \times 10^{-8} \ eV/T$$

eq^n ① can be rewrite as

$$\Rightarrow \mu_l = g_l \, l \cdot \mu_N \quad \text{——— ②}$$

where g_l is g vector associated with orbital angular momentum. For proton $g_l = 1$,

Since neutron donot have charge so, $g_l = 0$.

Protons and neutrons also have spin magnetic moments define by $\mu_s = g_s S \mu_N$ ——— ③

where $S = 1/2$, for protons and neutrons and g_s is $g_s = 5.585$ for proton & $g_s = -3.826$ for neutrons.

For proton and neutron $\mu_p = 5.585 \times \frac{1}{2} \mu_N$

$$\Rightarrow \mu_p = 2.792 \mu_N$$

and for neutron $\mu_n = -3.826 \times \frac{1}{2} \mu_N$

$$= -1.9128 \mu_N$$

Therefore, nuclear magnetic moment of different nuclei should be

$$\mu = Z \times \mu_p + (A - Z) \mu_N.$$

But experimentaly it is not like that, also cancellation of nuclear spin and spin magnetic moment occurs inside a nucleas. Thus all even even nuclei are observed to have zero magnetic moment. The magnetic observe for ^3_1H is $2.98 \mu_N$ [which is $\cong 2.79 \mu_N$] and for ^3_2He is $-2.1\ \mu_N$ ($\cong -1.91\ \mu_N$). But for $^{13}_6C$ the magnetic moment is $0.7\ \mu_N$, which is not equal to the expected value. Also it's positive. Similarly for $^{15}_7N$ the magnetic moment is $-0.28\ \mu_N$ which is not expected and also it's (-ve).

This is called carbon - Nitrogen (C-N) catastrophe. After this, the concept of orbital of the nucleons come. So the total magnetic moment the nucleus is nothing but the sum of orbital moment and spin moment.

The catastrophe result of 'C-N' can be solved by assuming that the nucleons participent in orbital motion. The magnetic moment is the vector sum of orbital moment & spin moment with $g_l = 1$ & $g_s = 5.585$ for proton & $g_l = 0$ and $g_s = -3.826$ for nutron.

Quadrupole moment of electrical

Atomic no. provides the charge of a nucleus it also provide the chemical property of a particular atom. It doesnot provide the distribution of charges. i.e. in which manner they are distributed. To know how the charges are distributed, electric quadrupole moment has to be known.

The variation of e⁻ field at a distance 'r' depends upon the distribution of charges. For spherical distri. the field varies as $\frac{1}{r^2}$, any distribution of e⁻ charge and current produce electric & magnetic field. The nature of the field produce at a distance is the characteristic of the distribution of electric charge & current. Each distribution of charge & current is assign with a corresponding multiple moment.

Nature of charge distribution	Nature of field varying.	corresponding multiple moment.
1 Spherical	$\frac{1}{r^2}$	monopole moment
2 Two equal & opposite charges separated by a distance	$\frac{1}{r^3}$	dipole moment
3 Two dipoles of elliptical charge distribution	$\frac{1}{r^4}$	Quadrupole moment

For monopole potential $V = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$,

The electric potential due to any charge distribution at a distance R, along Z axis $\longrightarrow$

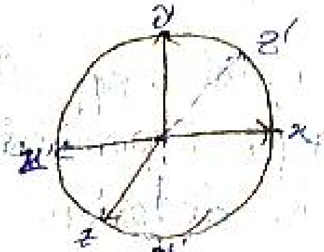
$$\Rightarrow V = \frac{1}{4\pi\epsilon_0}\left[\frac{1}{R}\int \rho\, dV + \frac{1}{R^2}\int \rho z\, dV + \frac{1}{R^3}\int \rho\left(3z^2 - r^2\right) dV + \cdots\cdots \right].$$

Here 1st term $\frac{1}{R}\int \rho \, dv$ is monopole moment 2nd is dipole & 3rd one is Quadrupole moment where ρ is the charge density.

$\boxed{\text{Note}}$ Quadrupole moment is zero for spherical or any other distribution. Only eliptical shape/distribution have quadrupole moment. Also for charge distribution Quadrupole moment zero.

For sphere :-

If x, y and z are the distance from the centre to the surface of a spherical nucleus



$$x^2 = y^2 = z^2$$

Therefor radius $r^2 = x^2 + y^2 + z^2$

$$r^2 = 3z^2$$

Quadrupole moment $= \int \rho \, (3z^2 - r^2) \, dv$

$$= 0$$

For spherical nucleus, if the particle moves in x, y plan then Quadrupole moment is equal to $-er^2$ (i.e. $z = 0$).

If the particle is moving along z axis i.e. $x = y = 0$

and $z^2 = r^2$

So, quadrupole moment is $Q = 2er^2$.

Nuclear Model:-   Chapter-2

① Liquid drop Model :-

Similarities between liquide drop model and a nucleas

① The attractive force near the nuclear surface is similar to the force of surface tension on the surface of a liquid drop.

② Density of nuclear matter does not depend on its vol^m. i.e. on it's mass number.

This suggest that saturation of nuclear force as in case of liquid drop.

The emission of particles like protons, nutron, alpha particle from the nucleas is similar to the emmision of molecule from the liquid drop during evaporation.

The formation of short lived compound nucleus during nuclear is similar to the process of conden-cesion from vapour to liquid. The internal energy of nucleas is similar to the heat energy within the liquid drop.

Bethe Weiszacker formula:-

(1) Volume Energy:-

Let us assume that the energy associated with each nucleon - nucleon bond is U and energy shared by each nucleon is $\frac{U}{2}$.

If we consider the spherical shape of the nucleons each interior nucleon has 12 others with each immidiat contect. Hence the interior has a binding energy $\frac{U}{2} \times 12 = 6U$

If there are A number of nucleons then the total binding energy of the nucleas is

$$E_v = 6\,UA$$

$$\Rightarrow E_v = a_1 A$$

$$\Rightarrow E_v \propto A$$

E_v is called vol^m energy of nucleas & it's directly proportional to A.

(2) Surface Energy:-

The nucleons which are on the surface of the nucleas have fewer than 12 neighbours.

The no. of such nucleons depends on the surface area of the nucleas. If the nucleus has radius R then the surface area is

$$= 4\pi R^2$$

$= 4\pi R_0^2 A^{2/3}$ [Since $R = R_0 A^{1/3}$]

Hence the no. of nucleons with fewer than 12 neighbours is proportional to $A^{2/3}$, which will reduce the total binding energy by $E_s = -a_2 A^{2/3}$. E_s is called surface energy of nucleus.

③ Coulomb Energy:—

If there Z no. of protons in the nucleus then the total no. of pairs of protons is equal to $\frac{Z(Z-1)}{2}$, Z^2 (of E_c) can be replaced by this

$$\Rightarrow E_c = \frac{1}{4\pi\epsilon_0} \frac{3}{5} \frac{Z(Z-1)e^2}{R_0 A^{1/3}}$$

The total binding energy will be reduced due to coulomb energy by an amount →

$$E_c = -a_3 \frac{Z(Z-1)}{A^{1/3}}$$

So, total binding energy of the nucleus is →

$$E_b = E_v + E_s + E_c$$

$$\Rightarrow E_b = a_1 A - a_2 A^{2/3} - a_3 \frac{Z(Z-1)}{A^{1/3}}$$

binding energy per nucleon is

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appearing
preparation

$$\Rightarrow \frac{E_b}{A} = a_1 - \frac{a_2}{A^{1/3}} - a_3 \frac{Z(Z-1)}{A^{4/3}}$$

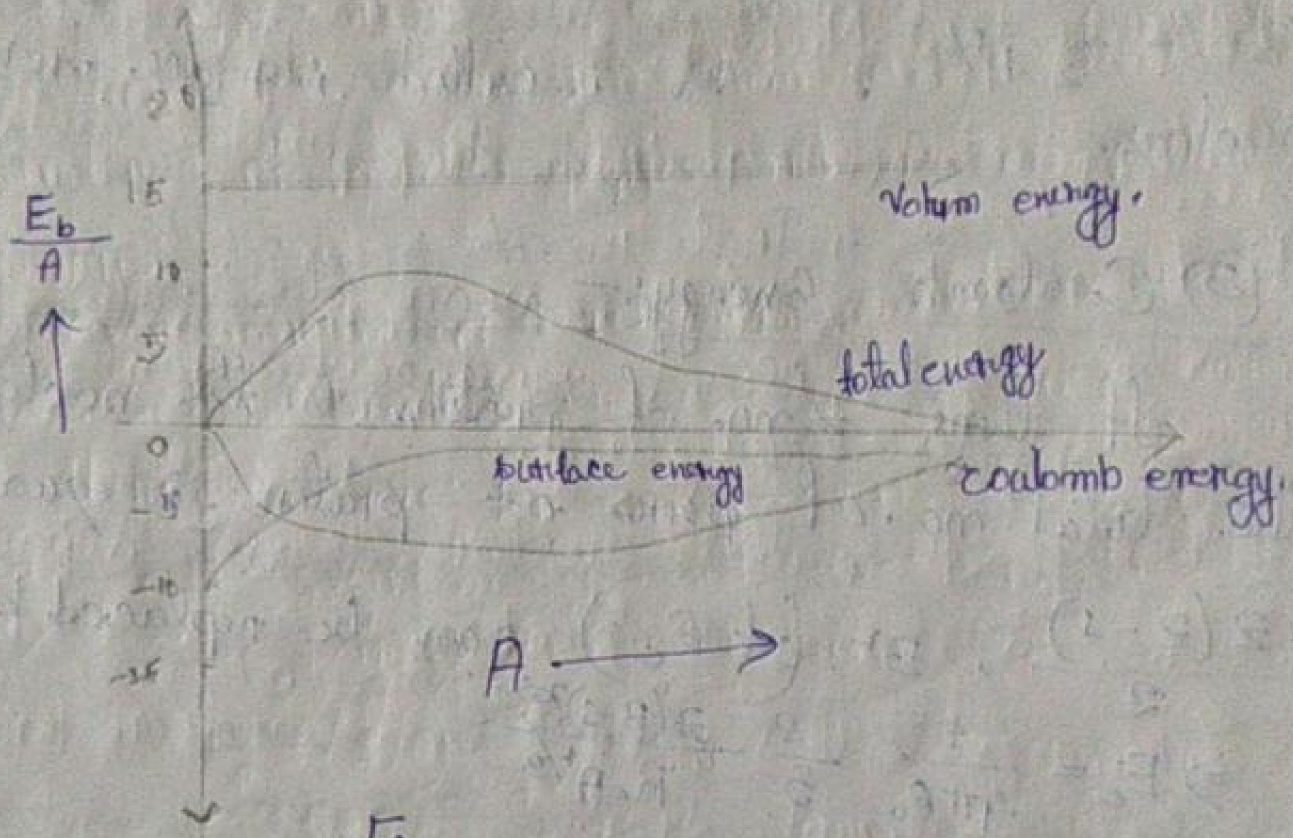

Fig :-

⊛ Corrections to the the semi-empirical mass formula :-

① Assymetry Energy :-

In the nuclei where no. of nutrons are long-er than the no. of protons higher energy levels have to be occupied then would be the case of nuclei where N & Z are equal.

Let us consider the energy levels of protons and nutrons with energy spacing ε.

In order to produce, for eg $N-Z=8$, without changing the mass no. nutrons would have to replace protons in an original nucleas, where $N=Z$.

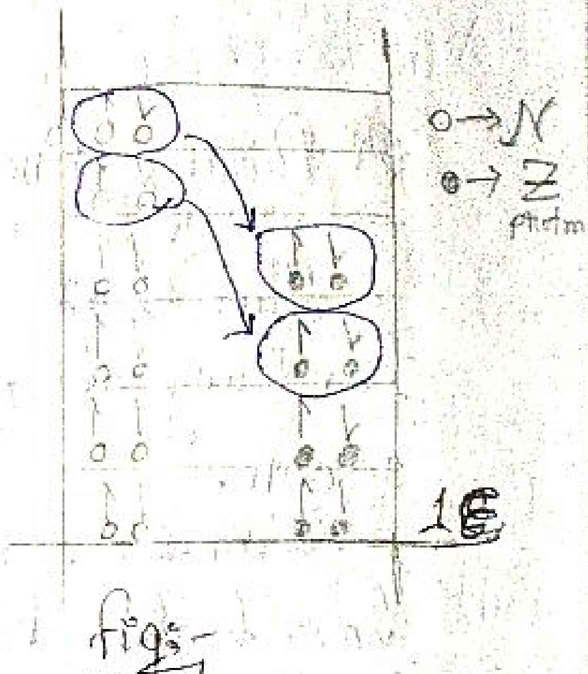

fig:-

The new nutrons occupie energy levels which is higher than 2ε of the proton they replaced. In general in case of $\frac{1}{2}(N-Z)$ new nutrons each nutrons must be reached by energy $\frac{1}{2}(N-Z)\frac{\varepsilon}{2}$.

The total energy will be

$$\Rightarrow \Delta E = (\text{no. of new nutron}) \times \left(\text{energy increas per nucleon}\right)$$

$$= \frac{1}{2}(N-Z) \times \frac{1}{2}(N-Z) \cdot \frac{\varepsilon}{2}$$

$$\Rightarrow \Delta E = \frac{\varepsilon}{8}(N-Z)^2$$

Since, $N = A - Z$

$$\text{So, } \Rightarrow \Delta E = \frac{\varepsilon}{8}(A-2Z)^2$$

The greater the no. of nucleons in a nucleas smaller is the energy spacing ε. i.e. $\varepsilon \propto \frac{1}{A}$.

Therefore the Assymetry energy E_a can be written as $E_a = -a_4 \frac{(A-2Z)^2}{A}$,

Pairing Energy:-

The last correction term arises from the tandency of proton pairs and nutron pairs to occur.

Even-Even nulcei are more stable & hence have higher binding energy. The pairing energy is therefore '+ve' for even-even nuclei, zero for odd-even nuclei and '-ve' for odd-odd nuclei. So pairing energy is given by

$$E_p = (\pm, 0)\frac{a_5}{A^{3/4}}$$

Semiempirical binding energy formula is

$$E_b = a_1 A - a_2 A^{2/3} - a_3 \frac{Z(Z-1)}{A^{1/3}} - a_4 \frac{(A-2Z)^2}{A}$$

$$(\pm, 0)\frac{a_5}{A^{3/4}}$$

The value of the coefficients are choosen as following in order to fit the experimental graph.

$a_1 = 14.1$ MeV

$a_2 = 13$ MeV

$a_3 = 0.595$ MeV

$a_4 = 19$ MeV

$a_5 = 13.5$ MeV

Q:- The atomic mass of $^{64}_{30}Zn$ is 63.929 u. Compare it's binding energy with the calculation from semiempirical binding energy formula?

Soln:- $B.E. = Zm_p + (A-Z)m_n - M_{atomic}\ ^A_Z X$

$\Rightarrow B.E. = 30 \times 1.007276 + (64-30) \times 1.00866 - 63.929$ u

$\Rightarrow B.E. = 30.21828 + 34.29444 - 63.929$ u

$\Rightarrow B.E. = 64.51272 - 63.929$ u

$\Rightarrow B.E. = 0.58372$ u

(A) Nuclear stability :-

Most stable nuclei in an isobar family:-

$$E_b = a_1 A - a_2 A^{2/3} - a_3 \frac{Z(Z-1)}{A^{1/3}} - a_4 \frac{(A-2Z)^2}{A}$$
$$(\pm, 0) \frac{a_5}{A^{3/4}}$$

For most stable nucleus E_b is maximum,

the $\frac{dE_b}{dZ} = 0$

So,

$$\Rightarrow Z = \frac{-a_3 A^{-1/3} + 4a_4}{2a_3 A^{-1/3} + 8a_4 A^{-1}}$$

1.190_1

$$\Rightarrow Z = \frac{-0.595 A^{-1/3} + 76}{1.19 A^{-1/3} + 152 A^{-1}}$$

For $A = 25$

$$\Rightarrow Z = \frac{-0.20349 + 76}{0.40698 + 152 \times (0.04)}$$

$$\Rightarrow Z = \frac{76.20349}{6.48698}$$

$\Rightarrow Z = 11.747$

which is approximately 12. Therefore $^{25}_{12}Mg$ is the most stable nucleus in the isobar family of $A = 25$.

Q 2:— Which isobar A = 75 does the liquid drop model suggest is the most stable?

Soln:— Given A = 75

Therefor

$$\Rightarrow Z = \frac{-0.595(75)^{-1/3} + 76}{2\times 0.595\,(75)^{-1/3} + 8\times 19\times (75)^{-1}}$$

$$\Rightarrow Z = \frac{-0.1410001 + 76}{0.28218 + 152\times 0.01334}$$

$$\Rightarrow Z = \frac{75.85891}{2.30885}$$

$$\Rightarrow Z = 32.86$$

which is approximately equal to 33 atomic no. of As (Arsenic).

Therefor Arsenic is most stable in its isobar family.

Q. 3:— Use the liquid drop model to establish which of the isobars $^{127}_{52}Te$ & $^{127}_{53}I$ decays into other and what kind of decay occurs?

Soln:—

(A) Mass Parabola :—

B.E. formula is

$$B.E. = Z m_p + (A-Z) m_n - M(A,Z)$$

$$M(A,Z) = Z m_p + N m_n - B.E \quad [\text{Since } A - Z = N]$$

then $E_b = a_1 A - a_2 A^{2/3} - a_3 \frac{Z(Z-1)}{A^{1/3}} - a_4 \frac{(A-2Z)^2}{A}$
$(\pm, 0) \frac{a_5}{A^{3/4}}$

Now considering $Z(Z-1)$ as Z^2 and value of B.E

$$\Rightarrow M(A,Z) = Z m_p + N m_n - a_1 A + a_2 A^{2/3} + a_3 \frac{Z^2}{A^{1/3}} + a_4 \frac{(A-2Z)^2}{A}$$

$$\Rightarrow M(A,Z) = F_A + P_Z + qZ^2 \quad \text{——— (1)}$$

There $F_A = A(m_n - a_1 + a_4) + a_2 A^{2/3}$ [not having Z components]

$$P_Z = -4a_4 - (M_n - m_p)$$

$$qZ^2 = \frac{1}{A}\left(a_3 A^{2/3} + 4a_4\right)$$

So, equation (1) is representing a parabola and it's called mass parabola.

From fig. (a) the lowest point of the parabola is the most stable nucleus of the given isobars.

All isobars having binding energy less than the most stable nucleus will lie on the either side of the most stable nucleus.

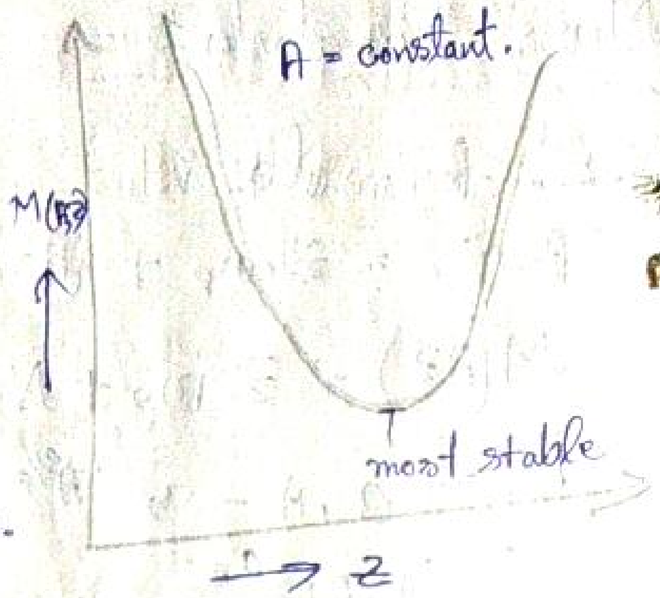

fig (a)

The isobars left to the most stable nucleus will emmit electrons (β^--decay). The isobars right to the most stable nucleus will emmit (β^+- decay) positrons.

Considering the pairing energy there will be two parabolas for even A. Pairing energy is '+ve' for even - even nuclei. '-ve' for odd-odd.

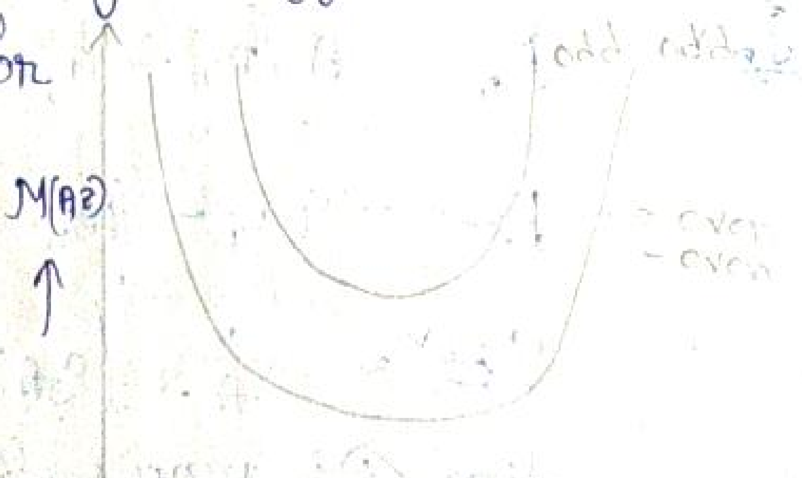

fig. (b)

Shell Model:-

Magic no:-

Atoms with 2, 10, 18, 36, 54 and 86 electrons have all their e^- shells completly fill; such electron structures have high binding energies and are exceptionally stable. The same kind is observed with respect to nuclei that have 2, 8, 20, 28, 50, 82 & 126 neutrons or protons are more abounded than other nuclei of similar mass numbers. Suggesting that their structure are more stable. These no. are known as magic numbers.

Evidences of shell Model:-

(1) Binding Energy:-

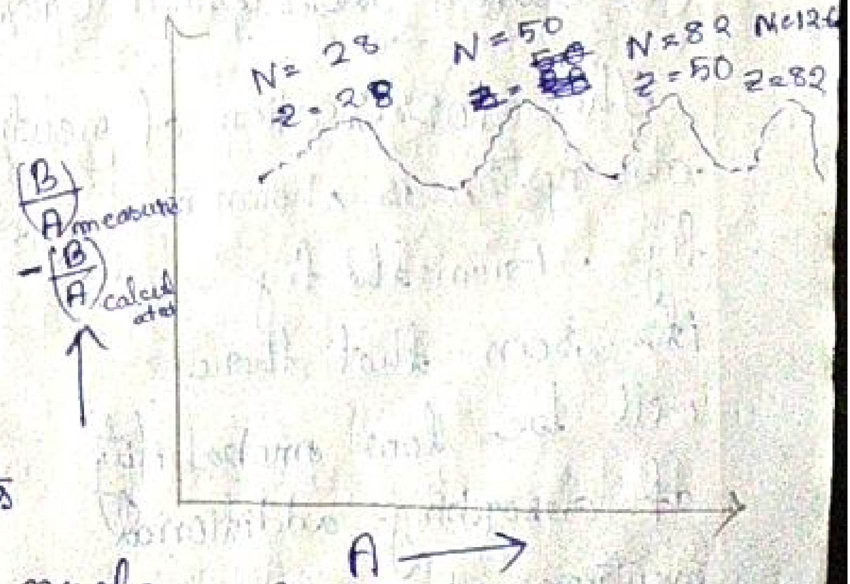

fig 1:-

Fig 1 shows the difference betn the liquid drop model predictions and actual measurments of binding energy per nucleon as a function of mass no. A.

There is evidence from the fig. 1. that there is excess binding energy for nuclei with N or Z equal to 28, 50, 82, 126 indicating more stability for these nuclei.

② No. of stable nuclides:-

As shown in fig. 2. larg number of stable nuclides are possible with N = 20, 28, 50, 82 indicating more stability of these nuclides.

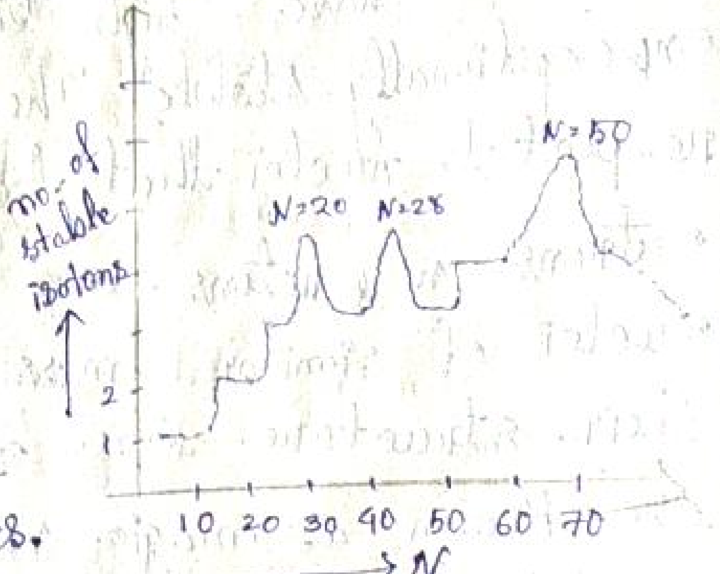

fig 2

③ Neutron absorption Cross-section:-

The cross-section of neutron absorption is shown in fig 3. From the fig it is seen that there will be low probability of absorbing additional neutron when N = 50, 82, 126.

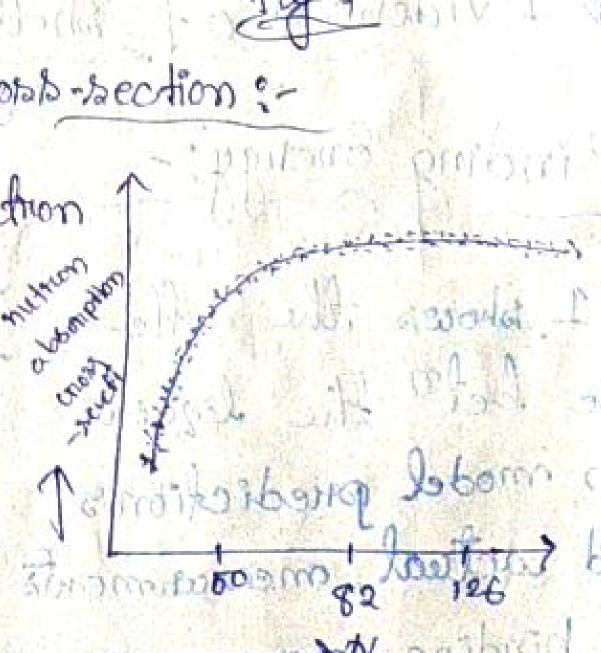

fig. 3

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④ The isotops in nature having nuclei containing magic no. of neutrons or protons have greater aboundence.

⑤ The electric Quadrupole moment of the nuclei having magic no. of protons/neutrons is zero, which indicates the spherical symmetry of the nucleus with closed shells.

④ How to reach magic numbers :-

The use of an appropriate potential in the schrodinger equation that could predict the magic no. The process simply as follows:-

① Approximate the mutual interaction betⁿ the neucleons by a single particle potential, called nuclear potential.

② Solve the schrodinger eqⁿ for the energy eigen state.

③ Use the results to determine the corresponding magic no. and compare with the experimantal no.

④ If the calculated results donot agree, revise the form of potential.

(A) Infinite Square Well Potential :-

This potential will account for the fact that the nucleons are well bound within the nucleus. The energy eigen values are obtained from the radial part of the schrodinger equation.

$$-\frac{\hbar^2}{2m}\frac{d^2U}{dx^2} + \left[V(r) + \frac{l(l+1)\hbar^2}{2mr^2}\right]U = EU \quad \text{--- (1)}$$

Where $l = 0, 1, 2, \ldots$

for infinite spherical square well, potential is defined as $V(r) = 0 \quad , r < R_0$

$$V(r) = \alpha \quad , r \gg R_0 .$$

In the region $r < R_0$

$$-\frac{\hbar^2}{2m}\frac{d^2U}{dx^2} + \frac{l(l+1)\hbar^2}{2mr^2}U = EU \quad \text{--- (2)}$$

The solⁿ of this eqⁿ (2) can be written as

$U(r) = r j_l(kr)$ · [Bessel's function] and

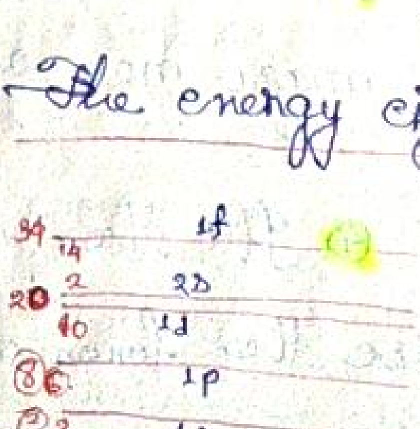
$$k = \sqrt{\frac{2mE}{\hbar^2}} .$$

Putting the boundary conditions ~~the~~ the energy eigen state can be found as follows

$l = 0 , \quad n = 1, 2, 3 \ldots$

$l = 1 , \quad n = 1, 2, 3 \ldots$

$l = 2 , \quad n = 1, 2, 3 , \ldots$



fig(1)

$2(2l+1)$ nucleons have been accumulated in the state having orbital angular momentum l. By considering this magic no. which arises in the shell closure are calculated and found that 2, 8, and 20 are similar with the experimental results, Other magic no. are not obtain.

(B) Harmonic Oscillator Potential:-

Harmonic Oscillator Potential is $V(r) = \frac{1}{2} m \omega^2 r^2$ putting this potential in radial part of the schrodinger equation:

$$\frac{-\hbar^2}{2m}\frac{d^2U}{dr^2} + \left[\frac{1}{2} m\omega^2 r^2 + \frac{l(l+1)\hbar^2}{2mr^2}\right]U = EU \quad \text{---} (3)$$

By solving this eqn (3).

the energy eigen values are found as follows $\rightarrow$

$$E = \left(2n + l + \frac{3}{2}\right)\hbar\omega, \quad \text{here } n = 0, 1, 2, \ldots$$

$$\Rightarrow E = \left(N + 3/2\right)\hbar\omega. \quad \boxed{\text{let } N = 2n + l}$$

The lowest energy state is

$E = 3/2\, \hbar\omega$ when $N = 0$, such $n = 0, l = 0$, 1s.

next $E = 5/2\, \hbar\omega$, when $N = 1$, $n = 0$, $l = 1$, 1p

next $E = 7/2\, \hbar\omega$, $N = 2$, such $n = 0$, $l = 1$, 1d

and $n = 1$, $l = 0$, 2s

next $E = \frac{9}{2}\hbar\omega$, $N = 3$, Such $n = 0$, $l = 3$, 1f

$n = 1$, $l = 1$, 2P

if $N = 4$,

$n = 0$, $l = 4$, 1g

$n = 1$, $l = 2$, 2d

$n = 2$, $l = 0$, 3s,

if $N = 5$, then $n = 0$, $l = 5$, 1h

$n = 1$, $l = 3$, 2f

$n = 2$, $l = 1$, 3p.

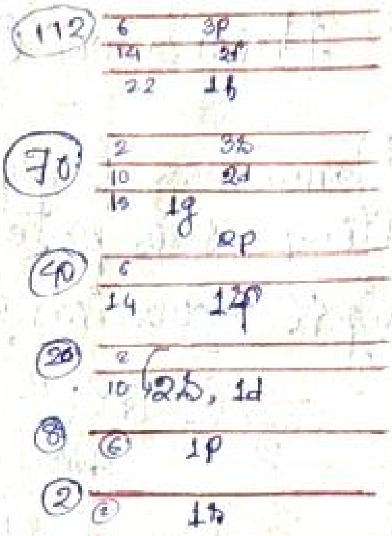


fig-2

The magic no. are not coming after 20 : considering the harmonic Oscillator potential.

(C) The magic no. cannot be describe by using Wood-saxon potential in the form $\left[a = 0.5\ \text{fm}\right]$

$$V = \frac{-V_0}{1 + \exp\left(\frac{r-R}{a}\right)}$$

(D) Spin-Orbit Potential :-

As the magic numbers cannot be explained by using different potential then scientist

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borrowed the idea of spin-orbit interaction of the nucleus and by incorporating it into the Hamiltonian.

The new ~~tot~~ hamiltonian in solving the energy eigen state will be $f(r)\langle \vec{l}\cdot\vec{s}\rangle + H_0 = H$

$$\boxed{\text{here } H_0 = \frac{-\hbar^2}{2m}\nabla^2 + V}$$

The total angular momentum after considering the spin-orbit interaction $\vec{J} = \vec{l} + \vec{s}$. As nucleon has spin ½ the possible values of total angular momentum quantum no. are

$$\vec{J} = (l + 1/2) \ \& \ (l - 1/2)$$

For $l = 0$, only one value it is $\vec{J} = 1/2$.

If square of $\vec{J}$

$$\vec{J}^2 = (\vec{l} + \vec{s})^2$$

$$= (\vec{l} + \vec{s}).(\vec{l} + \vec{s})$$

$$\Rightarrow \vec{J}^2 = \vec{l}^2 + 2\vec{l}\cdot\vec{s} + \vec{s}^2$$

$$\Rightarrow \langle \vec{l}\cdot\vec{s}\rangle = \frac{\vec{J}^2 - \vec{l}^2 - \vec{s}^2}{2}$$

for $\vec{J} = (l + 1/2)$

$$\langle \vec{l}\cdot\vec{s}\rangle = \frac{\vec{J}^2 - \vec{l}^2 - \vec{s}^2}{2}$$

$$= \frac{\vec{J}(\vec{J}+1)\hbar^2 - \vec{l}(\vec{l}+1)\hbar^2 - \vec{s}(\vec{s}+1)\hbar^2}{2}$$

$$= \frac{\hbar^2\left[\vec{J}(\vec{J}+1) - \vec{l}(\vec{l}+1) - \vec{s}(\vec{s}+1)\right]}{2}$$

$$= \frac{\hbar^2\left[(l+1/2)(l+3/2) - l(l+1) - 3/4\right]}{2}$$

$$= \frac{\hbar^2}{2}\left[\cancel{l^2}+2l+\cancel{3/4} - \cancel{l^2} - l - \cancel{3/4}\right]$$

$$\vec{J} = \frac{\hbar^2}{2} l.$$

For $\vec{J} = (\vec{l} - 1/2)$

$$\vec{J} = \frac{\hbar^2}{2}\left[(l-1/2)(l+1/2) - l^2 - l - 3/4\right]$$

$$\Rightarrow \vec{J} = \frac{\hbar^2}{2}\left[\cancel{l^2} - \frac{1}{4} - \cancel{l^2} - l - 3/4\right]$$

$$\Rightarrow \vec{J} = \frac{\hbar^2}{2}\left[-l-1\right]$$

$$\Rightarrow \vec{J} = \frac{-\hbar^2}{2}(l+1)$$

The value of $f(r)$ is '-ve' ∴ Therefore the larger value of $\vec{J}$ for a given state, $(l+1/2)$ will be passed downward. The splitting between $(l+1/2)$ and $(l-1/2)$ state is $\langle \vec{l}\cdot\vec{s}\rangle_{l+1/2} - \langle \vec{l}\cdot\vec{s}\rangle_{l-1/2}$.

$$= \frac{\hbar^2}{2} l + \frac{\hbar^2}{2}(l+1)$$

$$= \frac{\hbar^2}{2}(2l+1)$$

The energy splitting increases with increasing of 'l'. The splitting energy levels after incorporating spin-orbit interaction in the Hamiltonian in case of harmonic Oscillator potential are as follows → fig.

Therefore the magic no. can be properly explain by considering the spin-orbit potential.

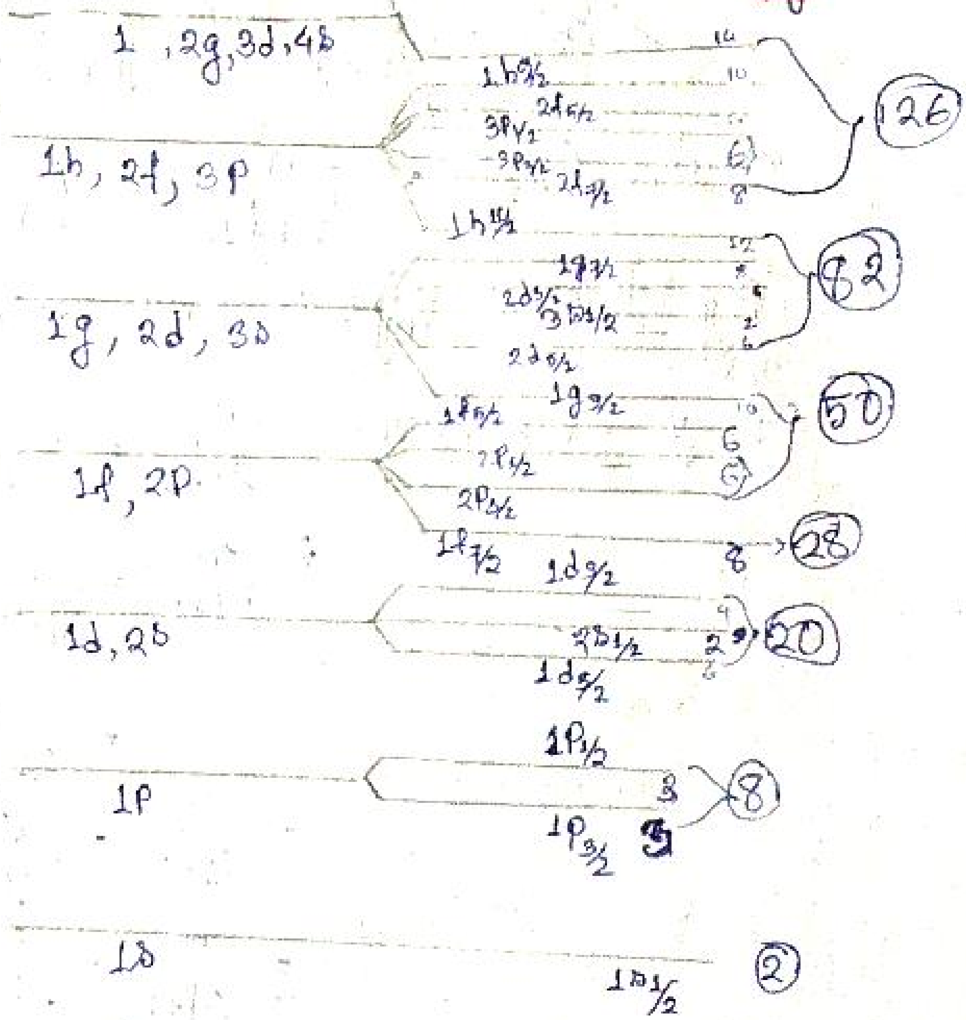

fig.

Magnetic moment using shell Models:-

z component of magnetic moment can be written as $\mu = \mu_N \left[g_l l_z + g_s s_z \right]$

$\Rightarrow \mu = \mu_N \left[g_l l_z + g_l s_z - g_l s_z + g_s s_z \right]$

$= \mu_N \left[g_l (l_z + s_z) + (g_s - g_l) s_z \right]$

$\Rightarrow \mu = \mu_N \left[g_l (\vec{J}_z) + (g_s - g_l) s_z \right]$

The expectation value of s_z is given by $\rightarrow$

$\langle s_z \rangle = \frac{m_j}{2l+1}$ for $\vec{J} = l + 1/2$

and $\langle s_z \rangle = \frac{-m_j}{2l+1}$ for $\vec{J} = l - 1/2$

By taking $m_j = j$ and expressing 'l' in terms of j we have $\frac{m_j}{2l+1} = \frac{j}{2(\vec{J} - \vec{s}) + 1}$ [for $j = l + 1/2$

$\Rightarrow \frac{m_j}{2l+1} = \frac{j}{2j - 2s + 1}$

$= \frac{j}{2j - 2 \times 1/2 + 1}$ [$s = 1/2$ for nucleons]

$\Rightarrow \frac{m_j}{2l+1} = \frac{j}{2j}$

$\Rightarrow \frac{m_j}{2l+1} = \frac{1}{2}$.

for $\vec{J} = l - 1/2$.

$\Rightarrow \frac{-m_j}{2l+1} = \frac{-j}{2j - 2 \times 1/2 + 1}$

$= -1/2 \cdot \frac{j}{j(j+1)}$

The expression for magnetic moment $\rightarrow$

$$\mu = \mu_N \left[g_l J + (g_s - g_l) s_z \right]$$

for $j = l + 1/2$

$$\Rightarrow \mu = \mu_N \left[g_l J + (g_s - g_l) \tfrac{1}{2} \right]$$

for $J = l - 1/2$

$$\Rightarrow \mu = \mu_N \left[g_l J + (g_s - g_l) \left(-\frac{1}{2} \frac{J}{J(J+1)} \right) \right]$$

* for unpaired proton :-

$$\Rightarrow \mu = \mu_N \left[J + 2.29 \right], \text{ for } j = l + 1/2$$

and $\mu = \mu_N \left[J - 2.29 \frac{J}{J+1} \right]$, for $J = l - 1/2$

* for unpaired neutron :-

$$\Rightarrow \mu = \mu_N \left[-1.91 \right], \text{ for } j = l + 1/2$$

and $\Rightarrow \mu = \mu_N \left[1.91 \frac{J}{J+1} \right]$, for $j = l - \frac{1}{2}$.

Q. Find out magnetic moment for $^{13}_{6}C$?

Soln :- here we know

$$\mu = \mu_N \, 1.91 \frac{J}{J+1}$$

$$= 1.91 \frac{1/2}{3/2} \mu_N \Rightarrow \mu = \frac{1.91}{3}$$

$\Rightarrow \mu = .63\,\mu_N$

its nearly equal to $\simeq 0.7$

Nuclear Force :-

The force that act between two or more nucleons and bind them into atomic nuclei is known as Nuclear force. A nuclear force is 10 millions times stronger than the chemical binding that holds atoms together.

Yukawa Meson Theory:-

The Japanese physicist Hideki Yukawa was the one who proposed that the interaction betn nucleons occur due to the exchange of some intermediat particles. These particles are termed as "Meson". The interaction between two nucleons N_1 and N_2 can be written as follows →

$$N_1 \rightleftharpoons N_2$$

$$\left.\begin{aligned} N_1 &\rightleftharpoons N_1 + \text{meson} \\ N_2 + \text{meson} &\rightleftharpoons N_2 \end{aligned}\right\} - (1)$$

and $N_1 \rightleftharpoons N_2 + \text{meson}$ ——— (2)

During this exchange proces there will be violation of the law of energy conservation by amount $\Delta E = mc^2$, where m is mass of the exchanged particle and this will be allowed by

Heisenburg uncertainity principle $\Delta E \times \Delta t = \hbar$

$$\Rightarrow \Delta E = \frac{\hbar}{\Delta t}$$

multiply & divided by c, $\Rightarrow \Delta E = \frac{\hbar c}{\Delta t c}$

$$\Rightarrow \Delta E = \frac{200 \text{ MeV}}{\text{Range bet}^n \text{ the nucleons}}$$

Let us consider the range bet^n the nucleons is 2 fm

$\therefore \Delta E = 100 \text{MeV}$.

So, The mass of the exchanged particle will be around $= \frac{100 \text{ MeV}}{c^2}$ i.e. $100 \text{ MeV}/c^2$ "

[The unet of nuclear mass is MeV/c^2 "]

This exchange process occur in a time 10^{-23} sec

The process in therms of π mesons" may be charged as π^+, π^- or π^0 neutral.

$S = vt$
$R = \Delta t c$
$\Delta t = \frac{R}{c}$

① Proton - Proton interaction $\Rightarrow P_1 - P_2$

$P_1 \longrightarrow P_1 + \pi^0$	$P_2 \longrightarrow P_1 + \pi^0$
$P_2 + \pi^0 \longrightarrow P_1$	$P_1 + \pi^0 \longrightarrow P_2$

② Proton - neutron, $P - n$

$P \longrightarrow P + \pi^0$	$n \longrightarrow n + \pi^0$
$P + \pi^0 \longrightarrow P$	$n + \pi^0 \longrightarrow n$

or,

$$p \longrightarrow n + \pi^{+}$$
$$n + \pi^{+} \longrightarrow p$$

$$n \longrightarrow p + \pi^{-}$$
$$p + \pi^{-} \longrightarrow n.$$

③ neutron – neutron $\Rightarrow n_1 - n_2$

$$n_1 \longrightarrow n_2 + \pi^{0}$$
$$n_2 + \pi^{0} \longrightarrow n_1$$

$$n_2 \longrightarrow n_1 + \pi^{0}$$
$$n_1 + \pi^{0} \longrightarrow n_2$$