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Simple harmonic motion: It is a periodic motion in which the particle oscillates on a straight line about a fixed point on the line, and the acceleration of the particle is always directed towards this fixed point while being magnitude is propotional to the displacement of the particle from the point.

Equation of motion of a simple harmonic motion:

The restoring force on the particle excecuting S.H.M.

$$F \propto -x$$

$$\Rightarrow F = -kx \quad , \quad k \text{ is the force or spring constant}$$

$$\Rightarrow m\frac{dv}{dt} = -kx$$

$$\Rightarrow \frac{dv}{dt} = -\frac{k}{m}x$$

$$\Rightarrow \frac{d^2x}{dt^2} = -\omega^2 x \quad , \text{ where } \omega^2 = \frac{k}{m}$$

$$\Rightarrow \frac{d^2x}{dt^2} + \omega^2 x = 0 \rightarrow (1)$$

[differential eqn of S.H.M.]

Multiplying equation (1) by $2\frac{dx}{dt}$

$$\Rightarrow 2\frac{dx}{dt}\frac{d^2x}{dt^2} + 2\frac{dx}{dt}\omega^2 x = 0$$

$$\Rightarrow \int\left(2\frac{dx}{dt}\frac{d^2x}{dt^2} + 2\frac{dx}{dt}\omega^2 x\right) = C \text{ , integrating both side.}$$

$$\Rightarrow \left(\frac{dx}{dt}\right)^2 + \omega^2 x^2 = C \rightarrow (2)$$

C is a constant of integration

Now, at maximum displacement of the particle i.e at $x = A$ (A is also called amplitude) the velocity $\frac{dx}{dt} = 0$. Putting this condition in equation (2)

$$0 + \omega^2 A^2 = C$$

$$\Rightarrow C = \omega^2 A^2 \rightarrow (3)$$

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From ② and ③

$$\left(\frac{dx}{dt}\right)^2 + \omega^2 x^2 = \omega^2 A^2$$

$$\Rightarrow \left(\frac{dx}{dt}\right)^2 = \omega^2 (A^2 - x^2)$$

$$\Rightarrow \frac{dx}{dt} = \pm\, \omega\sqrt{(A^2 - x^2)}$$

$$\frac{dx}{dt} = \omega\,(A^2 - x^2)$$

$$\Rightarrow \frac{dx}{\sqrt{A^2 - x^2}} = \omega\, dt$$

$$\Rightarrow \int \frac{dx}{\sqrt{A^2 - x^2}} = \omega \int dt$$

$$\Rightarrow \sin^{-1}\left(\frac{x}{A}\right) = \omega t \pm \phi$$

$$\Rightarrow x/A = \sin(\omega t \pm \phi)$$

$$\Rightarrow x = A \sin(\omega t \pm \phi)$$

$$\frac{dx}{dt} = -\omega\sqrt{(A^2 - x^2)}$$

$$\Rightarrow \frac{-dx}{\sqrt{A^2 - x^2}} = \omega\, dt$$

$$\Rightarrow \int \frac{-dx}{\sqrt{A^2 - x^2}} = \omega \int dt$$

$$\Rightarrow \cos^{-1}\left(\frac{x}{A}\right) = \omega t \pm \phi$$

$$\Rightarrow x/A = \cos(\omega t \pm \phi)$$

$$\Rightarrow x = A \cos(\omega t \pm \phi)$$

So $x = A \sin(\omega t \pm \phi)$ or $x = A\cos(\omega t \pm \phi)$ both are solutions of equation no. 1.

Assignment : Show that mechanical energy is conserved in S.H.M.